LINEAR ALGEBRA

Bachelor in Applied Mathematics BAM SEP-2023 LA-AM.1.S.A

Area Others

Number of sessions: 30

Academic year: 23-24

Degree course: FIRST

Number of credits: 6.0

Semester: 1º

Category: BASIC

Language: English

Professor: **IRENE ALDA**

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Irene Alda started to do research as a young physicist before and during her undergraduate studies. After completing her MSc. in Advanced Materials, she started a Ph.D. in Physics—technically in Photonics—at the Institute of Photonic Sciences (ICFO) with her own funding (FPU scholarship). Irene's research project was part of the ERC consolidator grant QnanoMECA in the group of ICREA Professor Dr. Romain Quidant. She graduated in 2020 "Cum Laude" with her dissertation: "Levitodynamics on-a-chip: from planar Paul traps to near-field optical nanocavities". During this time, she co-led teaching activities for undergraduate students from the *Universitat Politècnica de Catalunya*. In 2023, she joined IE University as the Academic Director of the Bachelor in Applied Mathematics. She ensures academic quality, excellent student experience, and effective management of the undergraduate program, plus, she is also involved in teaching activities.

Education:

- Ph.D in Photonics, ICFO & Universitat Politècnica de Catalunya.
- MSc. in Advanced Materials, Universidad Autónoma de Madrid.
- BSc. in Physics, Universidad Autónoma de Madrid.

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SUBJECT DESCRIPTION

In this course, you will explore the foundations of matrix theory and linear algebra, with a focus on practical applications in various fields. You will learn how to manipulate matrices, solve linear equation systems, and analyze linear transformations' properties. Throughout the course, you will work with real-world examples from physics, economics, social sciences, natural sciences, and engineering, giving you a deeper understanding of how linear algebra can be used to model and solve problems in these disciplines. By the end of the course, you will have gained the skills and knowledge needed to apply linear algebra to a wide range of real-world problems, making it an essential tool for any applied mathematician.

LEARNING OBJECTIVES

- Demonstrate knowledge and understanding of concepts related to vector spaces and vector subspaces (vector independence, bases, dimensions, intersection, and sum).
- Understand the concept of linear transformations and show proficiency in solving problems in linear transformations.
- Perform matrix computations and use matrix operations to efficiently solve linear systems (Gauss elimination) and determine the inverse of a matrix (e.g., Gauss Jordan, LU).
- Show proficiency in applying the properties of determinants when solving related problems.
- Demonstrate proficiency in finding eigenvalues and eigenvectors of a matrix and determining if a matrix is diagonalizable.
- Demonstrate knowledge and understanding of the Jordan form.
- Understand Singular Value Decomposition and applications in linear algebra.

TEACHING METHODOLOGY

Before each class, students are expected to work on assignments at home. The course lectures will cover both theoretical explanations and practical exercises. Each mathematical concept will be introduced or followed by one or more examples related to the students' fields of interest. It is important for students to participate in order to acquire the skills needed to solve exercises. Problem sets and brief quizzes will be used throughout the course to assess student progress.

Problem sets are in-depth problems that will be uploaded to Blackboard. It is strongly recommended that students complete the homework exercises during the course and not leave them for a date close to the exam. While students are encouraged to work with others on understanding the lecture material and attempting the regular assignments, problem sets are designed for individual work and are meant to provide feedback on how students are progressing.

Brief quizzes will be given throughout the semester, covering previously taught material. These quizzes are meant to test the overall understanding of the material and help the professor assess the overall performance and evolution of the class.

Learning Activity	**Weighting**	**Estimated time a student should dedicate to prepare for and participate in**
Lectures	26.67 %	40.0 hours
Discussions	16.67 %	25.0 hours
Exercises in class, Asynchronous sessions, Field Work	16.67 %	25.0 hours

Group work	13.33 %	20.0 hours
Individual studying	26.67 %	40.0 hours
TOTAL	100.0 %	150.0 hours

PREREQUISITES

You will benefit from a strong understanding of vectors, matrices, and three-dimensional coordinate systems. These foundational topics are essential to studying linear algebra, which involves manipulating and analyzing systems of linear equations and matrices. Throughout the course, you will build upon these concepts to explore more advanced topics and apply them to higher-dimensional spaces, including four-dimensional and n-dimensional spaces.

PROGRAM

Disclaimer: *The following description of the material covered is tentative. While an attempt will be made to cover all listed topics and include other advanced topics that will help students throughout their careers in applied mathematics, the pace of the classes will depend on group performance, which may introduce some variations in the syllabus.*

Pre and Post-work for each session:

- Necessary readings will be announced before the session.
- Selected exercises will be indicated at the end of each session.

The course is organized in the following modules:

- Vector spaces and subspaces
- Matrices and linear transformations
- Diagonalization
- Jordan form and Singular Value Descomposition

SESSION 1 (LIVE IN-PERSON)

Topics covered:

- Introduction to the course
- Introduction to LaTex

SESSION 2 (LIVE IN-PERSON)

Topics covered:

- Systems of linear equations
- Gaussian elimination

SESSION 3 (LIVE IN-PERSON)

Topics covered:

- Classification of systems of linear equations
- Rouché - Frobenius theorem

SESSION 4 (LIVE IN-PERSON)

Topics covered:

- Vector spaces and subspaces
- Linear combinations

SESSION 5 (LIVE IN-PERSON)

Topics covered:

- Linear dependence and linear independence
- Span of a set of vectors, intersections, and sum of vector spaces

SESSION 6 (LIVE IN-PERSON)

Topics covered:

- Bases and change of base as a linear system (introduction)
- Dimensions
- Steinitz theorem

SESSION 7 (LIVE IN-PERSON)

Topics covered:

- Linear transformations, kernel (or nullspace Ax=0) and image
- Isomorphisms
- Matrix representation of a linear transformation

SESSION 8 (LIVE IN-PERSON)

Topics covered:

- Matrix multiplication as the composition of transformations (A=CR)
- The matrix "zoo" and operations with matrices

SESSION 9 (LIVE IN-PERSON)

Topics covered:

- Inverse of a matrix (Gauss Jordan)
- LU decomposition

SESSION 10 (LIVE IN-PERSON)

Review + Quiz

SESSION 11 (LIVE IN-PERSON)

Topics covered:

- Linear equations in matrix representation (Ax=b)
- Change of basis (definition and formula)

SESSION 12 (LIVE IN-PERSON)

Topics covered:

- Applications in linear algebra

SESSION 13 (LIVE IN-PERSON)

Topics covered:

- Properties of determinants

SESSION 14 (LIVE IN-PERSON)

Topics covered:

- Determinant formulas & cofactors
- Cramer's rule and inverse matrix

SESSION 15 (LIVE IN-PERSON)

Review for midterm exam

SESSION 16 (LIVE IN-PERSON)

Midterm exam

SESSION 17 (LIVE IN-PERSON)

Topics covered:

- Eigenvalues
- Eigenvectors
- Characteristic polynomial

SESSION 18 (LIVE IN-PERSON)

Topics covered:

- Problems on finding eigenvalues and eigenvectors

SESSION 19 (LIVE IN-PERSON)

Topics covered:

- More on diagonalization
- Powers of A

SESSION 20 (LIVE IN-PERSON)

Topics covered:

- Invariant subspaces
- The Caley-Hamilton Theorem

SESSION 21 (LIVE IN-PERSON)

Topics covered:

- Problems on diagonalization and Cayley-Hamilton theorem

SESSION 22 (LIVE IN-PERSON)

Topics covered:

- Minimal polynomial
- Algebraic and geometric multiplicity

SESSION 23 (LIVE IN-PERSON)

Topics covered:

- Applications in linear algebra

SESSION 24 (LIVE IN-PERSON)

Review + Quiz

SESSION 25 (LIVE IN-PERSON)

Topics covered:

- The Jordan form: definition and examples

SESSION 26 (LIVE IN-PERSON)

Topics covered:

- Exercises on the Jordan form

SESSION 27 (LIVE IN-PERSON)

Topics covered:

- Singular Value Decomposition

SESSION 28 (LIVE IN-PERSON)

Topics covered:

- Applications in linear algebra

SESSION 29 (LIVE IN-PERSON)

Review for final exam

SESSION 30 (LIVE IN-PERSON)

Final exam

EVALUATION CRITERIA

criteria	percentage	Learning Objectives	Comments

Final Exam	30 %		It is worth 30% of the overall grade. You need to score at least 3.5 on the final exam to pass the overall course, even if you have already passed the course through the other course assessments.
Intermediate tests	30 %		The mid-term exam will take place on Session 15, and it is worth 30% of the overall grade (jointly with other intermediate tests/quizzes).
Individual work	30 %		Each module has its own problem set including several exercises. The student is expected to work on them and to turn in selected exercises that will be announced throughout the course.
Class Participation	10 %		Students are expected to participate actively during lectures with questions and remarks. Class grade will be based also on punctuality, participation, and class conduct. There may be a penalty if you create a disruption or talk excesively during class.

RE-SIT / RE-TAKE POLICY

Each student has four chances to pass any given course distributed over two consecutive academic years: ordinary call exams and extraordinary call exams (re-sits) in June/July.

Students who do not comply with the 80% attendance rule during the semester will fail both calls for this Academic Year (ordinary and extraordinary) and have to re-take the course (i.e., re-enroll) in the next Academic Year.

Evaluation criteria:

- Students failing the course in the ordinary call (during the semester) will have to re-sit the exam in June / July (except those not complying with the attendance rule, who will not have that opportunity and must directly re-enroll in the course on the next Academic Year).
- The extraordinary call exams in June / July (re-sits) require your physical presence at the

campus you are enrolled in (Segovia or Madrid). There is no possibility to change the date, location or format of any exam, under any circumstances. Dates and location of the June / July re-sit exams will be posted in advance. Please take this into consideration when planning your summer.

- The June / July re-sit exam will consist of a comprehensive exam. Your final grade for the course will depend on the performance in this exam only; continuous evaluation over the semester will not be taken into consideration. Students will have to achieve the minimum passing grade of 5 and can obtain a maximum grade of 8.0 (out of 10.0) – i.e., “notable” in the in the re-sit exam.
- Retakers: Students who failed the subject on a previous Academic Year and are now re-enrolled as re-takers in a course will be needed to check the syllabus of the assigned professor, as well as contact the professor individually, regarding the specific evaluation criteria for them as retakers in the course during that semester (ordinary call of that Academic Year). The maximum grade that may be obtained in the retake exam (3rd call) is 10.0

After ordinary and extraordinary call exams are graded by the professor, you will have a possibility to attend a review session for that exam and course grade. Please be available to attend the session in order to clarify any concerns you might have regarding your exam. Your professor will inform you about the time and place of the review session. Any grade appeals require that the student attended the review session prior to appealing.

Students failing more than 18 ECTS credits after the June / July re-sits will be asked to leave the Program. Please, make sure to prepare yourself well for the exams in order to pass your failed subjects.

In case you decide to skip the opportunity to re-sit for an exam during the June / July extraordinary call, you will need to enroll in that course again for the next Academic Year as a re-taker and pay the corresponding extra cost. As you know, students have a total of four allowed calls to pass a given subject or course, in order to remain in the program

BIBLIOGRAPHY

Recommended

- Gilbert Strang. *Introduction to Linear Algebra.* ISBN 9781733146678 (Digital)
- Stephen Friedberg, Arnold Insel, Lawrence Spence. *Linear Algebra.* ISBN 0134860241 (Digital)
- W. Keith Nicholson. *Linear Algebra with Applications.* ISBN 1717015220 (Digital) https://lyryx.com/wp-content/uploads/2018/01/Nicholson-OpenLAWA-2018A.pdf
- Peter J. Olver, Chehrzad Shakiban. *Applied Linear Algebra.* ISBN 9783319910413 (Digital)
- Mike Cohen. *Practical Linear Algebra for Data Science: From Core Concepts to Applications Using Python.* ISBN 1098120612 (Digital)
- Mike X Cohen. *Linear Algebra: Theory, Intuition, Code.* Sincxpress Education SRL. ISBN 9083136604 (Digital)

BEHAVIOR RULES

Please, check the University's Code of Conduct here. The Program Director may

provide further indications.

ATTENDANCE POLICY

Please, check the University's Attendance Policy here. The Program Director may provide further indications.

ETHICAL POLICY

Please, check the University's Ethics Code here. The Program Director may provide further indications.