GEOMETRY

Bachelor in Applied Mathematics BAM SEP-2023 G-AM.1.S.A

Area Others

Number of sessions: 30

Academic year: 23-24

Degree course: FIRST

Number of credits: 6.0

Semester: 1º

Category: BASIC

Language: English

Professor: **ALEXANDRE ANAHORY**

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ALEXANDRE ANAHORY

Alexandre Anahory graduated in Physics at University of Lisbon and obtained a Ph.D. in Mathematics from Universidad Autónoma de Madrid. He has conducted research at the Instituto de Ciencias Matemáticas in Madrid and at the Center for Automation and Robotics, also in Madrid.

His research focus on the application of modern cutting-edge mathematical theories to diverse areas such as physics, engineering, robotics, computer science, among others.

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SUBJECT DESCRIPTION

This course, along with linear algebra, aims to cover the fundamentals of linear algebra and geometry, with a primary focus on Euclidean geometry. These courses cover concepts such as vector spaces and linear transformations, matrix operations, inner product spaces, and the Euclidean group. In addition to traditional instruction in this field, there is also an emphasis on computational aspects and implementing these operations using software libraries. The course will also cover applications in statistics and data science.

LEARNING OBJECTIVES

This course aims to improve students' understanding of: Euclidean geometry, scalar and vector product, norms and metric spaces, calculation of angles and distances, isometries and Euclidean group, bilinear and quadratic forms, orthogonality, and affine spaces. After this course, students will also be able to implement computational methods that apply transformations to rigid 3D objects (translations, rotations, inversions, scale changes, etc.).

- **Module 0: Introduction to geometry**: Familiarization with the main concepts appearing frequently in geometry: vectors, distances, lines, curves, planes.
- **Module 1: Euclidean space:** Concept of dot product. Concept of Euclidean space. Matrix expression of a dot product. Properties. Norm induced by a dot product. Properties. Distance between vectors. Angle between vectors. Orthogonality between vectors. Gram-Schmidt orthogonalization process. Orthogonal linear variety.
- **Module 2: Bilinear forms and quadratic forms:** Concept of bilinear form. Analytical expression and matrix expression of a bilinear form. Concept of quadratic form. Matrix expression of a quadratic form. Classification of quadratic forms. Positive-definite and negative-definite matrices. Spectral theorem.
- **Module 3: Movements and similarities:** Previous concepts: Euclidean affine space (distance), isometry (properties), affine mapping (matrix expression). Concept of movement. Direct and inverse movements. Properties. Concept of a fixed point. Classification of movements in the plane. Matrix expression. Concept of similarity. Matrix expression.

TEACHING METHODOLOGY

IE University teaching method is defined by its collaborative, active, and applied nature. Students actively participate in the whole process to build their knowledge and sharpen their skills. Professor's main role is to lead and guide students to achieve the learning objectives of the course. This is done by engaging in a diverse range of teaching techniques and different types of learning activities such as the following:

Learning Activity	**Weighting**	**Estimated time a student should dedicate to prepare for and participate in**
Lectures	40.0 %	60.0 hours
Discussions	3.33 %	5.0 hours
Exercises in class, Asynchronous sessions, Field Work	30.0 %	45.0 hours
Group work	10.0 %	15.0 hours
Individual studying	16.67 %	25.0 hours
TOTAL	100.0 %	150.0 hours

PROGRAM

SESSION 1 (LIVE IN-PERSON)

Introduction to the course. Introduction to geometry and examples of applications where geometry is used.

SESSION 2 (LIVE IN-PERSON)

Review of vectors in two and three dimensions: vector addition, multiplication by scalar, norm. Dot and vector product of two vectors. Properties of dot products, angle between vectors and norm of a vector. Review of operations with complex numbers.

SESSION 3 (LIVE IN-PERSON)

Distance between two points. Properties of the exterior product and the signed area of the parallelogram formed by two vectors. Vector expressions of a line, a plane and a curve. Complex representation of a curve in the plane.

SESSION 4 (LIVE IN-PERSON)

Conic curves in two dimensions: ellipse, hyperbola and parabola. Notable applications where one finds conics. Introduction to basic software to visualize points, vectors, distances, planes, curves and conics.

SESSION 5 (LIVE IN-PERSON)

Practice Class.

SESSION 6 (LIVE IN-PERSON)

Notions of coordinate geometry. Discussion on the meaning of coordinates. Change of coordinates. Comparison of different distances in the two-dimensional plane and applications to a set of real-world problems.

SESSION 7 (LIVE IN-PERSON)

Euclidean Space and the matrix expression of an inner product. Norms and distances associated with inner products.

SESSION 8 (LIVE IN-PERSON)

Orthogonality of two vectors. Orthogonal projections. Gram-Schmidt algorithm.

SESSION 9 (LIVE IN-PERSON)

Practice Class.

SESSION 10 (LIVE IN-PERSON)

Orthogonal complements of Lines and Planes in two and three dimensions.

SESSION 11 (LIVE IN-PERSON)

Transformations between vector spaces. Isometries of Euclidean Spaces. Rotations and Translations of vectors in two and three-dimensions.

SESSION 12 (LIVE IN-PERSON)

Introduction to linear and affine transformations between vector spaces.

SESSION 13 (LIVE IN-PERSON)

Practice Class.

SESSION 14 (LIVE IN-PERSON)

Bilinear forms. Matrix expression of a bilinear form. Symmetric and skew-symmetric bilinear forms.

SESSION 15 (LIVE IN-PERSON)

Quadratic forms: matrix expression and classification of quadratic forms.

SESSION 16 (LIVE IN-PERSON)

Wrap-up session.

SESSION 17 (LIVE IN-PERSON)

Mid-term exam.

SESSION 18 (LIVE IN-PERSON)

Principal axis and diagonalization of Quadratic forms.

SESSION 19 (LIVE IN-PERSON)

Practice Class.

SESSION 20 (LIVE IN-PERSON)

Affine Euclidean spaces: operations, varieties and distance. Orthogonality of two varieties.

SESSION 21 (LIVE IN-PERSON)

Affine maps and associated linear maps. Euclidean motions: notable examples.

SESSION 22 (LIVE IN-PERSON)

Euclidean motions in coordinates: the Euclidean group.

SESSION 23 (LIVE IN-PERSON)

Practice Class.

SESSION 24 (LIVE IN-PERSON)

The glide vector and classification of Euclidean motions.

SESSION 25 (LIVE IN-PERSON)

Similar euclidean motions, invariance levels and fixed points.

SESSION 26 (LIVE IN-PERSON)

Classification of euclidean motions of the Line and the Plane in Space.

SESSION 27 (LIVE IN-PERSON)

Practice Class.

SESSION 28 (LIVE IN-PERSON)

Advanced Topic: Introduction to non-Euclidean geometries. Elliptic and hyperbolic geometries.

SESSION 29 (LIVE IN-PERSON)

Wrap-up session.

SESSION 30 (LIVE IN-PERSON)

Final Exam.

EVALUATION CRITERIA

A. Class participation

Class participation will be evaluated based on the quality of your questions, remarks and answers. The students are expected to engage in class activity and to demonstrate interest for the course.

B. Problem Sets

At the end of each chapter, the students will be given a problem set to assess their progression. Some lectures will be dedicated to discuss the problem sets and some exercises will be due as homework.

C. Quizzes, Exercises, and Other Activities

During some sessions, you will be given a short quiz based on the content taught on the previous sessions. Quizzes will be announced in advance and can be either computer-based or paper-based and are typically composed of a few questions, aimed at ensuring that every student has a basic understanding of the material covered in class in the previous sessions.

D. Final exam

There will be one final exam. In order to pass the course, you need a minimum grade of 3.5 in the final exam. If your grade in the final exam does not reach the threshold value of 3.5, you will fail the course, even in the case in which your weighted average (computed using the table above) exceeds 5.0.

criteria	percentage	Learning Objectives	Comments
Final Exam	30 %		
Quizzes	30 %		
Problem Sets	30 %		
Participation	10 %		

RE-SIT / RE-TAKE POLICY

Each student has four chances to pass any given course distributed over two consecutive academic years: ordinary call exams and extraordinary call exams (re-sits) in June/July.

Students who do not comply with the 80% attendance rule during the semester will fail both calls for this Academic Year (ordinary and extraordinary) and have to re-take the course (i.e., re-enroll) in the next Academic Year.

Evaluation criteria:

•Students failing the course in the ordinary call (during the semester) will have to re-sit the exam in June / July (except those not complying with the attendance rule, who will not have that opportunity and must directly re-enroll in the course on the next Academic Year).
• The extraordinary call exams in June / July (re-sits) require your physical presence at the campus you are enrolled in (Segovia or Madrid). There is no possibility to change the date, location or format of any exam, under any circumstances. Dates and location of the June / July re-sit exams will be posted in advance. Please take this into consideration when planning your summer.
• The June / July re-sit exam will consist of a comprehensive exam. Your final grade for the course will depend on the performance in this exam only; continuous evaluation over the semester will not be taken into consideration. Students will have to achieve the minimum passing grade of 5 and can obtain a maximum grade of 8.0 (out of 10.0) – i.e., “notable” in the in the re-sit exam.
• Retakers: Students who failed the subject on a previous Academic Year and are now re-enrolled as re-takers in a course will be needed to check the syllabus of the assigned professor, as well as contact the professor individually, regarding the specific evaluation criteria for them as retakers in the course during that semester (ordinary call of that Academic Year). The maximum grade that may be obtained in the retake exam (3rd call) is 10.0.

After ordinary and extraordinary call exams are graded by the professor, you will have a possibility to attend a review session for that exam and course grade. Please be available to attend the session in order to clarify any concerns you might have regarding your exam. Your professor will inform you about the time and place of the review session. Any grade appeals require that the student attended the review session prior to appealing.

Students failing more than 18 ECTS credits after the June-July re-sits will be asked to leave the Program. Please, make sure to prepare yourself well for the exams in order to pass your failed subjects.

In case you decide to skip the opportunity to re-sit for an exam during the June / July extraordinary call, you will need to enroll in that course again for the next Academic Year as a re-taker and pay the corresponding extra cost. As you know, students have a total of four allowed calls to pass a given subject or course, in order to remain in the program.

BIBLIOGRAPHY

Recommended

- Strang. (2016). *Introduction to Linear Algebra.* 5. SIAM. ISBN 9780980232776 (Printed)

This provides an advanced introduction to linear algebra, and is accompanied by a set of video lectures of the author, available at: https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/

- Howard Anton & Chris Rorres. (2014). *Elementary Linear Algebra with Supplemental Applications.* 11th. Wiley. ISBN 9781118677452 (Printed)

Elementary Linear Algebra 11th edition gives an elementary treatment of linear algebra that is suitable for a first course for undergraduate students. The aim is to present the fundamentals of linear algebra in the clearest possible way; pedagogy is the main consideration. Calculus is not a prerequisite, but there are clearly labeled exercises and examples (which can be omitted without loss of continuity) for students who have studied calculus.

- Audin, M.. *Geometry..* Berlin: Springer Verlag. ISBN 3540434984 (Digital)
- Reventós, Agustí. *Affine maps, euclidean motions and quadrics.* Springer. ISBN 9780857297 (Digital)
- Lay, Lay, McDonald. (2021). *Linear Algebra and Its Applications.* 6th. Pearson. ISBN 9781292351216 (Printed)

This book provides a modern elementary introduction to linear algebra.

BEHAVIOR RULES

Please, check the University's Code of Conduct here. The Program Director may provide further indications.

ATTENDANCE POLICY

Please, check the University's Attendance Policy here. The Program Director may provide further indications.

ETHICAL POLICY

Please, check the University's Ethics Code here. The Program Director may provide further indications.