CALCULUS 1

Bachelor in Applied Mathematics BAM SEP-2023 C1-AM.1.S.A

Area Others

Number of sessions: 30

Academic year: 23-24

Degree course: FIRST

Number of credits: 6.0

Semester: 1º

Category: BASIC

Language: English

Professor: **MIGUEL VAQUERO VALLINA**

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Miguel Vaquero received his Licenciatura and Master's degrees (w. honors) in mathematics from the Universidad de Santiago de Compostela, Spain. He moved to Madrid to get his PhD in applied mathematics from ICMAT (center of excellence, Severo Ochoa Distinction) publishing in reputed journals in the field and spending time abroad visiting renowned universities in North America. During that time he also taught at the Universidad Autónoma de Madrid. In 2017 he joined the research group of J. Cortés at the University of California, San Diego in the Mechanical and Aerospace Engineering Department. His recent research efforts focused mainly on the design of new algorithms able to cope with challenging problems from physics and engineering using techniques from dynamical systems and optimization in a data-driven fashion.

Experience

Postdoctoral Research Scholar, University of California, San Diego (2017-2019)

Postdoctoral Research Scholar, ICMAT, Madrid (2016-2017)

Teaching Assistant, Universidad Autónoma de Madrid

Education

Ph.D in Mathematics, ICMAT.

MSc in Mathematics, Universidad de Santiago de Compostela

Bs in Mathematics, Universidad de Santiago de Compostela

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SUBJECT DESCRIPTION

In this course the student is introduced to one of the main branches of mathematics: Calculus. Calculus relies on the idea of limit to construct derivatives and integrals. It would be impossible to overstate the importance of these concepts in science and engineering. Think about it, anything related to velocity, rate of change, length or area is based on these ideas. Thus, Calculus will pave the way to study some of the most fascinating mathematical models in forthcoming subjects, like Ordinary Differential Equations, Nonlinear Dynamics or Optimization.

The student will be presented with a blended approach, where the theoretical and practical aspects are introduced and connected with real world examples, and illustrated aided by computers.

LEARNING OBJECTIVES

The main objective of this course is to set the foundations of calculus and analytical thinking, and to develop the necessary abstract reasoning and computational tools that the students will use during the rest of their trajectory. At the end of the course, students should be able to:

- Use abstract reasoning.
- Re-formulate real-world problems in mathematical language.
- Master the the key concepts of mathematical analysis: limit, derivative, integral...
- Make complex computations involving derivatives and integrals.
- Solve problems in optimization.

Additionally, the course will help in the acquisition or reinforcement of generic skills:

- The ability to summarize and present information in a meaningful and accurate way.
- The ability to isolate the core questions from the more superfluous ones.
- The ability to quickly identify the tools that need to be used in a particular problem.

TEACHING METHODOLOGY

IE University teaching method is defined by its collaborative, active, and applied nature. Students actively participate in the whole process to build their knowledge and sharpen their skills. Professor's main role is to lead and guide students to achieve the learning objectives of the course. This is done by engaging in a diverse range of teaching techniques and different types of learning activities such as the following:

Learning Activity	**Weighting**	**Estimated time a student should dedicate to prepare for and participate in**
Lectures	33.33 %	50.0 hours
Discussions	16.67 %	25.0 hours

Exercises in class, Asynchronous sessions, Field Work	16.67 %	25.0 hours
Group work	16.67 %	25.0 hours
Individual studying	16.67 %	25.0 hours
TOTAL	100.0 %	150.0 hours

PROGRAM

Disclaimer: *The following description of the material covered is tentative. An attempt will be made to cover all listed topics and to include other advanced topics that will help the student throughout his/her career in computer science. However, the pace of the classes will depend on group performance, which may introduce some variations in the syllabus.*

The course is divided in 4 modules described below.

- **MODULE 1:** LIMITS AND CONTINUITY
- **MODULE 2:** SEQUENCES AND SERIES
- **MODULE 3:** DIFFERENTIAL CALCULUS IN ONE VARIABLE
- **MODULE 4:** INTEGRATION OF FUNCTIONS OF ONE VARIABLE

SESSION 1 (LIVE IN-PERSON)

Topics treated:

- Introduction to the subject. What? Why? How?
- Introduction to advanced mathematics.
- Computational resources and coding mathematics.

Pre and Post-work:

- No pre-work needed.
- After the session problem sets and instructions about how to work with them will be available online.

SESSION 2 (LIVE IN-PERSON)

Topics treated:

- Concept of limit, intuitive idea and formal definition..
- Definition of lateral limits.
- Techniques for computing limits. Brief intro to continuous functions, implications towards the existence of limit. First theorems about limits: sum, difference, constant multiple, product, quotient, power, root. Factor and cancel technique.
- Examples and applications of limits.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 3 (LIVE IN-PERSON)

Topics treated:

- Advanced techniques for computing limits.
- Rigorous definitions of limit.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 4 (LIVE IN-PERSON)

Topics treated:

- Infinite limits: Vertical Asymptote.
- Limits at infinity: Horizontal Asymptote.
- Examples.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 5 (LIVE IN-PERSON)

Topics treated:

- Continuity.
- Continuity of composite functions.
- Bolzano's Theorem (Intermediate Value Theorem).
- Examples and applications.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 6 (LIVE IN-PERSON)

SESSION 6 (LIVE IN-PERSON)

- Big O and Little o notation. Applications to algorithmic complexity.
- Work in groups to understand key examples

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 7 (LIVE IN-PERSON)

Review of Module + Quiz.

SESSION 8 (LIVE IN-PERSON)

Topics treated:

- Concept of sequence.
- Definition of limit of a sequence.
- Techniques for computing limits of sequences.
- Examples and applications.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 9 (LIVE IN-PERSON)

Topics treated:

- Concept of sequence (cont).
- Definition of limits of a sequence (cont).
- Techniques for computing limits of sequences (cont).
- Examples and applications of sequences (cont).
- Recurrence relations.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 10 (LIVE IN-PERSON)

Topics treated:

- Concept os series.
- Limit of series.
- Divergent series.

Pre and Post-work:

- Necessary readings will be announced before the session.

- Work on selected exercises.

SESSION 11 (LIVE IN-PERSON)

Topics treated:

- Geometric and p-harmonic series.
- Convergence and divergence criteria. Root, Comparison, Limit Comparison, Divergence Test.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 12 (LIVE IN-PERSON)

Topics treated:

- Geometric and p-harmonic series (cont.).
- Convergence and divergence criteria. Root, Comparison, Limit Comparison, Divergence Test (cont.).

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 13 (LIVE IN-PERSON)

Review of module + Quiz.

SESSION 14 (LIVE IN-PERSON)

Review for the Midterm. In this session we will go over all the content covered and work together on selected exercises designed to practice for the midterm exam.

SESSION 15 (LIVE IN-PERSON)

Midterm

SESSION 16 (LIVE IN-PERSON)

Topics treated:

- Rigorous definition of derivative.
- Derivative as a limit.
- Derivative as a rate of change.
- Derivative as the slope of a tangent.

- Derivative as a function.
- Examples and applications.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 17 (LIVE IN-PERSON)

Topics treated:

- Methods for computation of derivatives.
- Numerical approximation.
- Symbolic computations and necessary software.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 18 (LIVE IN-PERSON)

Topics treated:

- Higher-order derivatives.
- Concept of convexity and concavity. Characterization using the second-order derivative.
- Use of the first-order and second-order derivatives to find the stationary points of a function and classify them.
- Examples and applications.
- Optimization algorithms for functions of one variable. Implementation.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 19 (LIVE IN-PERSON)

Topics treated:

- What does the derivative and higher-order derivatives tell us about a function?
- Graphical representation of functions. We will use the concepts developed until this session to plot and understand functions.
- Examples and applications.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 20 (LIVE IN-PERSON)

Topics treated:

- Graphical representation of functions. We will use the concepts developed until this session to plot and understand functions (cont.).
- Examples and applications (cont.).

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 21 (LIVE IN-PERSON)

In this session you will work in groups some examples to understand the following theorems and their rigorous proofs.

- Mean Value Theorem.
- L'Hopital's rule.

Examples and applications of these theorems will be introduced.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 22 (LIVE IN-PERSON)

Topics treated:

- Taylor's polynomial.
- Applications of differential calculus in real life.
- We wil show some interesting applications of differential calculus, illustrating and deepening in the concepts introduced so far.

Pre and Post-work:

- Necessary readings will be announced before the session.
- Work on selected exercises.

SESSION 23 (LIVE IN-PERSON)

Topics:

- Antiderivative and indefinite integral. Definite integral and net area.
- Fundamental Theorem of Calculus. Proof.
- Examples and applications.

Pre and Post-work:

- Necessary readings will be announced before the session.
- After the session a new problem set will be available.

SESSION 24 (LIVE IN-PERSON)

Topics:

- Antiderivative and indefinite integral. Definite integral and net area (cont.).
- Fundamental Theorem of Calculus. Proof (cont.).
- Examples and applications (cont.).

Pre and Post-work:

- Necessary readings will be announced before the session.
- After the session a new problem set will be available.

SESSION 25 (LIVE IN-PERSON)

Topics treated:

- Advanced methods of integration.
- Numerical approximation of definite integrals (Simpson's and Trapezoidal Rules)
- Implementation of numerical integration algorithms.

Pre and Post-work:

- Necessary readings will be announced online before the session.
- Work on selected exercises.

SESSION 26 (LIVE IN-PERSON)

In this session you will work in groups some applications of differential calculus in real life.

- Computation of area and net area.
- Computation of length of a curve.
- Computation of area surface of revolution.
- Implementation of related algorithms.

Pre and Post-work:

- Necessary readings will be announced online before the session.
- Work on selected exercises.

SESSION 27 (LIVE IN-PERSON)

In this session you will work in groups some applications of differential calculus in real life.

- Solving simple ordinary differential equations.
- Applications to physics and other fields.

Pre and Post-work:

- Necessary readings will be announced online before the session.
- Work on selected exercises.

SESSION 28 (LIVE IN-PERSON)

Review of module + Quiz.

SESSION 29 (LIVE IN-PERSON)

Review for the final exam. In this session we will work on selected exercises to review all the previous content for the final exam.

SESSION 30 (LIVE IN-PERSON)

Final Exam. Remember the final exam is worth 30% of your final grade. You will need at least a 3.5 in the final exam to pass the course.

EVALUATION CRITERIA

criteria	percentage	Learning Objectives	Comments
Final Exam	30 %		It is worth 30% of the overall grade. You need to score at least 3.5 on the final exam to pass the overall course, even if you have already passed the course through the other course assessments. Information about the detailed characteristics of the final-exam will be given at the beginning of the semester.

Intermediate tests	30 %		The mid-term exam will take place on Session 15, and it is worth 30% of the overall grade (jointly with other intermediate test). Information about the detailed characteristics of the mid-term exam will be given at the beginning of the semester.
Individual work	30 %		Each module has its own problem set including several exercises. The student is expected to work on them and to turn in selected exercises that will be announced throughout the curse. Each problem set will be graded after the deadline and your overall "Homework" grade is worth 30% of the final grade. The “Homework” grade will be the arithmetic mean of all the problem sets' marks, excluding the worst score.
Class Participation	10 %		It will be worth 10% of the overall grade. Students are expected to participate actively during lectures with questions and remarks. Class grade will be based also on punctuality, participation, and class conduct. There may be a penalty if you create a disruption or talk excesively during class.

RE-SIT / RE-TAKE POLICY

Each student has four chances to pass any given course distributed over two consecutive academic years: ordinary call exams and extraordinary call exams (re-sits) in June/July.

Students who do not comply with the 70% attendance rule during the semester will fail both calls for this Academic Year (ordinary and extraordinary) and have to re-take the course (i.e., re-enroll) in the next Academic Year.

Evaluation criteria:

•Students failing the course in the ordinary call (during the semester) will have to re-sit the exam in June / July (except those not complying with the attendance rule, who will not have that opportunity and must directly re-enroll in the course on the next Academic Year).
• The extraordinary call exams in June / July (re-sits) require your physical presence at the campus you are enrolled in (Segovia or Madrid). There is no possibility to change the date, location or format of any exam, under any circumstances. Dates and location of the June / July re-sit exams will be posted in advance. Please take this into consideration when planning your summer.
• The June / July re-sit exam will consist of a comprehensive exam. Your final grade for the course will depend on the performance in this exam only; continuous evaluation over the semester will not be taken into consideration. Students will have to achieve the minimum passing grade of 5 and can obtain a maximum grade of 8.0 (out of 10.0) – i.e., “notable” in the in the re-sit exam.
• Retakers: Students who failed the subject on a previous Academic Year and are now re-enrolled as re-takers in a course will be needed to check the syllabus of the assigned professor, as well as contact the professor individually, regarding the specific evaluation criteria for them as retakers in the course during that semester (ordinary call of that Academic Year). The maximum grade that may be obtained in the retake exam (3rd call) is 10.0.

After ordinary and extraordinary call exams are graded by the professor, you will have a possibility to attend a review session for that exam and course grade. Please be available to attend the session in order to clarify any concerns you might have regarding your exam. Your professor will inform you about the time and place of the review session. Any grade appeals require that the student attended the review session prior to appealing.

Students failing more than 18 ECTS credits after the June-July re-sits will be asked to leave the Program. Please, make sure to prepare yourself well for the exams in order to pass your failed subjects.

In case you decide to skip the opportunity to re-sit for an exam during the June / July extraordinary call, you will need to enroll in that course again for the next Academic Year as a re-taker and pay the corresponding extra cost. As you know, students have a total of four allowed calls to pass a given subject or course, in order to remain in the program.

BIBLIOGRAPHY

Recommended

- L. Goldstein, D. Lay, D. Schneider, N. Asmar,. (2017). *Calculus and its applications..* 14th. Pearson. ISBN 9780134437774 (Digital)
- W. Briggs, L. Cochran, B. Gillett, E. Schulz. (2018). *Calculus: Early Transcendentals.* 3rd. Pearson. ISBN 9780134763644 (Digital)

BEHAVIOR RULES

Please, check the University's Code of Conduct here. The Program Director may

provide further indications.

ATTENDANCE POLICY

Please, check the University's Attendance Policy here. The Program Director may provide further indications.

ETHICAL POLICY

Please, check the University's Ethics Code here. The Program Director may provide further indications.